

On the Cartan involution on co-adjoint orbits of a compact semi-simple Lie group.

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It is well-known that complex and real projective spaces $\mathbb{C}P^n$ and $\mathbb{R}P^n$ have isomorphic cohomology rings with \mathbb{Z}_2 coefficients, when, for $\mathbb{C}P^n$, degrees are halved. We say that the complex conjugation τ on $X = \mathbb{C}P^n$ is of *real type*, and that $X^\tau = \mathbb{R}P^n$ is a *real core* for τ . The same is known to happen with the complex conjugation on Grassmannians and other (partial) flag manifolds. In this talk, we generalize these results by showing that the Cartan involution on any (co-)adjoint orbit of any semi-simple compact Lie group G is of real type (the case $G = SU(n)$ gives the above mentioned flag manifolds with the complex conjugation). The equivariant cohomology plays a major role, as well as the Hamiltonian action of the maximal torus of G on the co-adjoint orbits.