Straightening Lemma Revisited

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Abstract. Let G be a *compact* Lie group and let $K \supset H$ be closed subgroups of G, and denote by M_π the G-space obtained from the mapping cylinder of the canonical projection $\pi : G/H \to G/K$. The group $ET(M_\pi)$ of equivariant translations of M_π, effectively $\frac{N(H)\cap N(K)}{H}$, is central (and hence normal) in $\mathbf{Homeo}^G_{M_\pi}(M_\pi, M_\pi)$. The *Straightening Lemma*, see [Bre72], V.4.1, states that $ET(M_\pi)$ is a strong deformation retract of $\mathbf{Homeo}^G_{M_\pi}(M_\pi, M_\pi)$ in compact-open topology.

The presentation discusses the following modifications to the Straightening Lemma. First, we upgrade G to any real or complex Banach Lie group. Second, we let $K_0 \supset \cdots \supset K_n$ be compact subgroups of G and replace M_π by the equivariant simplex $\Delta_n(G; K_0, \ldots, K_n)$ (below X) which is found in many works of Sören Illman.

We show that $\mathbf{Homeo}^G_{X/G}(X, X)$ is in fact a topological group and that $ET(X)$, now effectively $\frac{N(K_0)\cap\cdots\cap N(K_n)}{K_n}$, is a strong $ET(X)$-deformation retract of $\mathbf{Homeo}^G_{X/G}(X, X)$. In particular,

$$\mathbf{Homeo}^G_{X/G}(X, X)/ET(X)$$

is a contractible topological group.

References

[Bre72] G. E. Bredon, *Introduction to compact transformation groups*, Academic Press, New York, 1972, Pure and Applied Mathematics, Vol. 46.