G-Morse Theory Revisited

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We consider smooth closed G-manifolds with G a compact Lie group and discuss the concepts of *G-Morse pair* and *generalized G-triangulation*.

A G-Morse pair is a pair (h, g) consisting of a G-invariant Riemannian metric and h a G-Morse function whose gradient w.r.to g satisfies a strong transversality condition.

For any critical orbit Σ of h there are two well defined (up to equivalency) representations $\rho^{\Sigma}_{\pm} : H \to GL(V_{\pm})$ with $H \subset G$ the orbit type of Σ.

A generalized G-triangulation is a Morse-Smale pair (h, g) so that for any critical orbit Σ the second representation, ρ^{Σ}_{-}, is trivial.

The first main result (known but without a satisfactory proof) is about a G-Morse pair and claims that the space of trajectories between two critical orbits and the stable and unstable manifolds of each critical orbit can be canonically compactified to smooth G-manifolds with corners and these corners are explicitly described.

Standard topology results in conjunction with the present proof imply that a G-Morse pair gives rise to a smooth *G-handle body*, while a generalized G-triangulation to a smooth G-simplicial triangulation in the sense of Illman and others.

The second main result claims is that any G-manifold admits G-generalized triangulations (half of this result is due to W. Meyer) and any smooth G-triangulation gives rise to a G-generalized triangulation in an essentially canonical way.

An other important application of the compactification theorem (the first result mentioned above) is a G-version of Witten Helffer Sjöstrand theory considerably more sophisticated than the "nonequivariant" one which does permit, among others, to understand G-analytic torsion and compare it to the topological torsion (G-Reidemeister torsion).